A Study of the Hippocampal Network via Persistent Homology

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Background

In the neuroscience community, it is believed that place cells (PC) in an animal's hippocampus play a central role in forming mental maps, which are used by the animal for spatial navigation. Place cells are sensitive to location, in the sense that each place cell becomes active when the animal enter a specific area, called the cell's place field (PF), in an environment. We study the following question: **Given time series data showing the activation record of a sufficiently large number of place cells as an animal explores an arena, can we infer some topological properties of the arena?** By employing techniques from **Persistent Homology (PH)** to study the network of cells we are able to accurately recover the number of obstacles in an arena that the animal has explored.

Introduction

We simulate a set of random walks of a rat in arenas with different numbers of obstacles which are randomly placed. To generate the place cells' activation records, we used a Poisson firing model. Then each activation record was transferred into a **network**, which can be interpreted as the transition matrix of a Markov Chain with each node representing a place cell. For each network we computed a set of network invariants, called **persistence diagrams**, using the theory of persistent homology. At last, the differences between each pair of diagrams, where each diagram corresponds to a separate trial, was computed using the *bottleneck distance* [1].

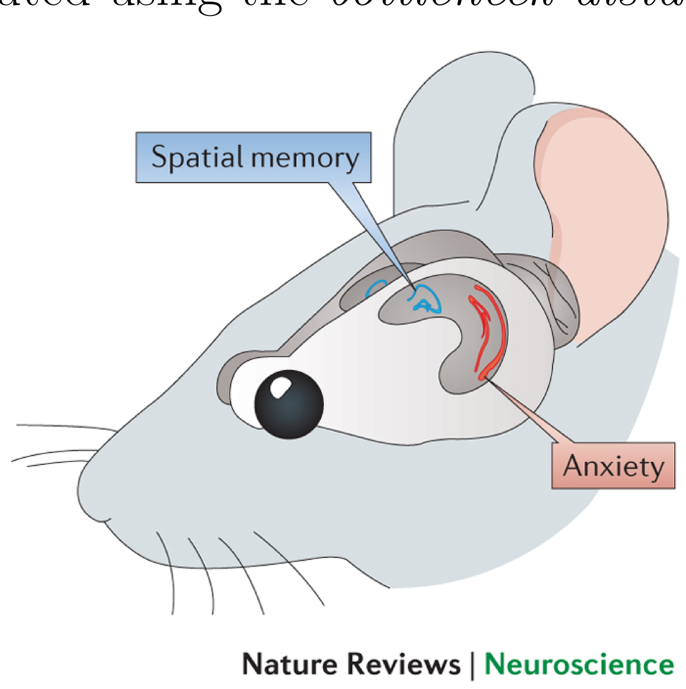
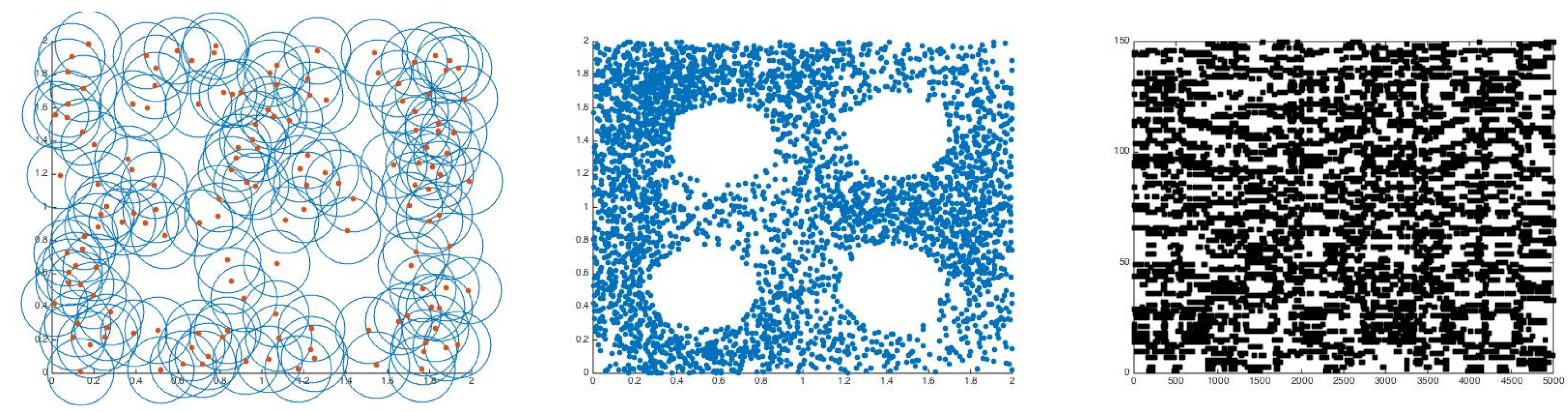


Figure 1: Rodent hippocampus



Variables	
Arena size	$2m \times 2m$
Radius of obstacle	$25cm$
Number of PF	150
Max radius of PF	$15cm$
Max firing rate	$20hz$
Number of Steps	5000
Speed per step	15cm

Figure 2: PF and PC, Trajectory and Spike train of a 4 obstacle arena with Setups

Simulation Procedure

First generate 5 arenas $\{A_i | i \in 0, 1 \cdots 5\}$ by randomly choosing i places to put in obstacles away from each other and the arena's boundary. Second uniformly randomly assign 150 PFs 25 times for each A_i getting $\{A_{ij} | j \in 1, 2 \cdots 25\}$ covering the whole arena except obstacles.

Then simulate rat's random walk by randomly choosing a direction each step and move along that direction if next step is not out of boundary or in a obstacle, getting time series rat location data R. The mean μ of Poisson firing model of each place cell c at step t is defined as follows:

$$\mu = f \times \exp\left(\frac{1}{2}\left(\frac{dist(c, R(t))}{s}\right)^2\right) \tag{1}$$

Here f is generated by a log normal distribution whose mean is the max firing rate r and standard deviation $1.2r$, and $dist(c, R(t))$ is the distance between c and position $R(t)$ of the rat at time t. This process outputs the spike train (raster). The total number of spike trains was **125**.

Raster to Network

A network is a set of nodes that are connected by a set of edges. Each edge could have a weight.
For each spike train, a network (X, ω_X) is constructed as follows: X consisted of 150 nodes (one per PC), and for each $1 \le i, j \le 150$, the weight of edge (x_i, x_j) is given by $\omega_x(x_i, x_j) = 1 - \frac{N_{i,j}(5)}{N_j}$, where

$$\begin{aligned} N_{i,j}(5) &= \#\text{ time cell } x_j \text{ spiked over threshold in a window of} \\ &\quad \text{5 steps after cell } x_i \text{ spiked over threshold} \\ N_j &= \sum_j N_{i,j} \end{aligned} \tag{2}$$

Homology

Homology is a mathematical tool to find topological features (loops) of a shape. A *persistence diagram* is a set of intervals representing the topology of the space, parametrized by resolution. It is obtained by converting data into a family of topological objects called simplicial complexes, organizing them by an inclusion hierarchy called a filtration, and then applying the theory of PH to it. In this study we use *Dowker persistent homology of networks* [1].

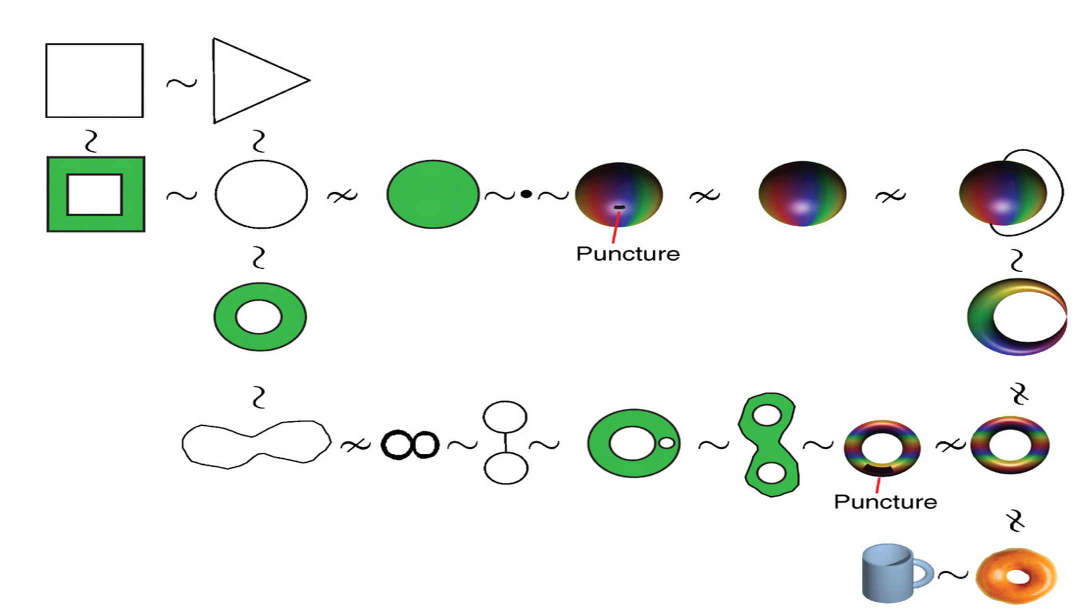


Figure 3: Loop structures

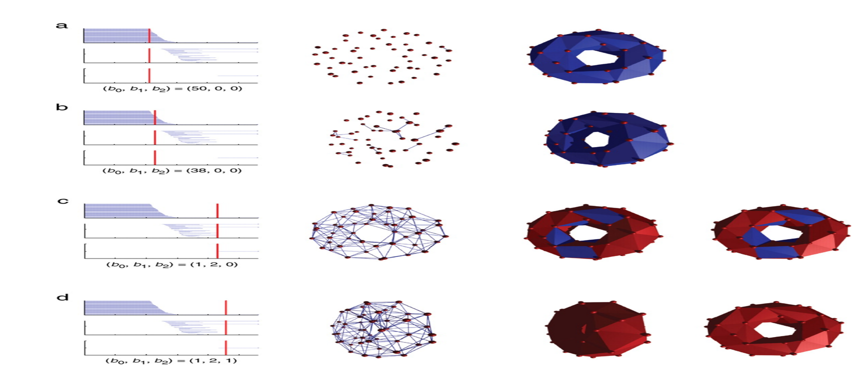


Figure 4: Persistent Diagram [2]

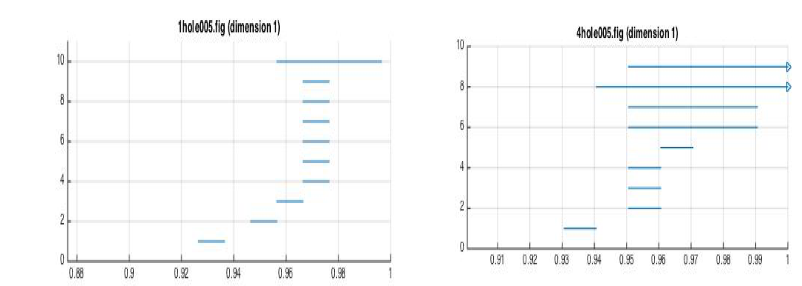


Figure 5: Persistence diagrams of arenas with 1 and 4 obstacles

Conclusion

The dissimilarity between the 111 barcodes was represented as a single linkage dendrogram. Our results show that diagrams corresponding to arenas with the same number of obstacles are clustered together, and are far apart from arenas with different numbers of obstacles. This means *Dowker complex* can catch all topological feature of an arena.

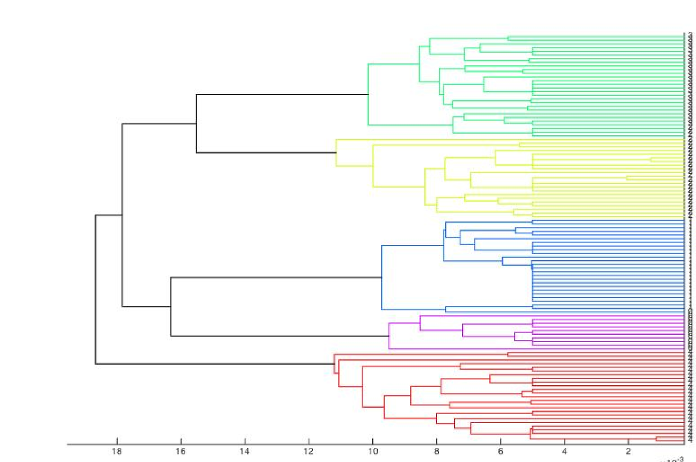


Figure 6: Single Linkage Dendrogram

References

[1] S. Chowdhury and F. Mémoli, "Persistent homology of directed networks," in *50th Asilomar Conference on Signals, Systems, and Computers, page to appear, IEEE*, 2016.

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[3] G. Singh, F. Memoli, T. Ishkhanov, G. Sapiro, G. Carlsson, and D. L. Ringach, "Topological analysis of population activity in visual cortex," *Journal of vision*, vol. 8, no. 8, pp. 11–11, 2008.

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Acknowledgements

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